Model Answer
Mid 2014

1 - Signal Manipulation

A Even and odd parts

I. $x_1(n) = u(n)$

II. $x_2(n) = a^n u(n)$.

Sol

(a) $x_{even}(n) = \frac{1}{2}[x(n) + x(-n)]$

$= \frac{1}{2}[u(n) + u(-n)] = \begin{cases} 1 & n=0 \\ \frac{1}{2} & n \neq 0 \end{cases}$ (2)

$= \frac{1}{2} + \frac{1}{2}\delta(n)$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$= \begin{cases} \frac{1}{2} & n>0 \\ 0 & n=0 \\ -\frac{1}{2} & n<0 \end{cases} = \frac{1}{2}\,sgn(n)$.

(b) $x_e(n) = \frac{1}{2}[a^n u(n) + a^{-n}u(-n)] = \begin{cases} \frac{1}{2}a^n & n>0 \\ 1 & n=0 \\ \frac{1}{2}a^{-n} & n<0 \end{cases}$ (2)

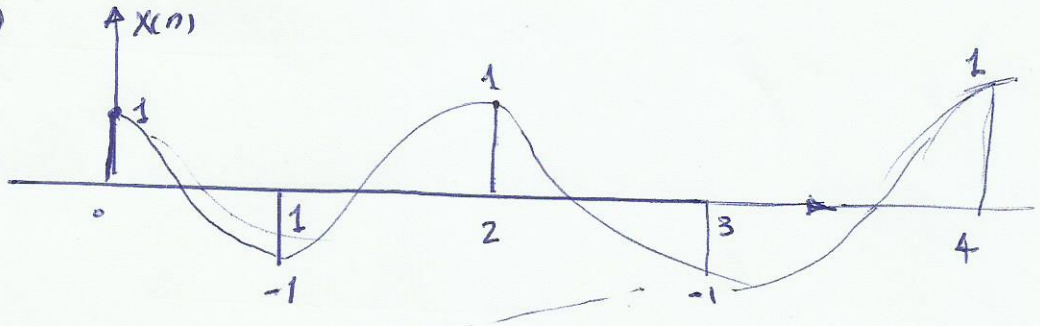
$= \frac{1}{2}a^{|n|} + \frac{1}{2}\delta(n)$

$x_o(n) = \frac{1}{2}[a^n u(n) - a^{-n}u(-n)] = \frac{1}{2}a^{|n|}\,sgn(n)$.

(B) [2]

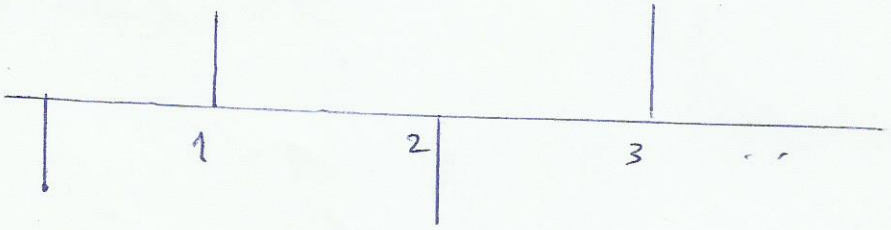
a. $F_N = 2F_m = 20\,kHz$ (1)

b. (1)



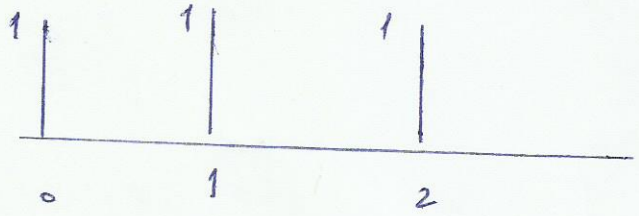
(c) Delayed 3

(1)



(d) compressed by 2

(1)



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[2] (A) check linearity

(2)

if $x_1(n) = c\,x(n)$

$\therefore y_1(n) = 5c\,x(n+2) + 2c\,x(n+1) + 3c\,x(n+4)$

which is not the same as

$e\,y(n) = c\,[5x(n+2) + 2x(n+1) + 3x(n) + 1]$

$\therefore$ sys is not homogeneous.

If $x(n) = x_1(n) + x_2(n)$

$$\therefore y(n) = 5[x_1(n+2) + x_2(n+2)] + 2[x_1(n+1) + x_2(n+1)] + 3[x_1(n) + x_2(n)] + 1$$

which is not the same as

$y_1(n) + y_2(n)$

$$= 5[x_1(n+2) + x_2(n+2)] + 2[x_1(n+1) + x_2(n+1)] + 3[x_1(n) + x_2(n)] + 2$$

(2)

$\therefore$ sys not additive

$\therefore$ sys not linear.

(B) $x(n) = x(n) + x(n-1) + x(n-2)$ (2)

check sys. shift-Invarient

$$y(n-n_0) = x(n-n_0) + x(n-n_0-1) + x(n-n_0-2)$$

$$y_1(n) = x_1(n) + x_1(n-1) + x_1(n-2)$$

$n \rightarrow n-n_0$

$$\therefore y_1(n) = x_1(n-n_0) + x_1(n-n_0-1) + x_1(n-n_0-2)$$

$\therefore$ sys shift invarient

3) A

Convolve

$$x(n) = 2\cos\left(\frac{n\pi}{2}\right)[u(n) - u(n-4)]$$

& $h(n) = 0.5\,n\,[u(n) - u(n-3)]$

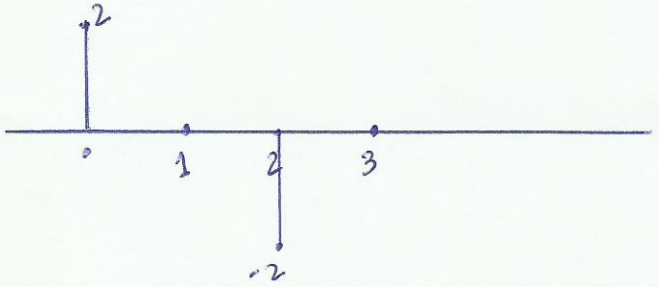
$x(n)$ valid from $0 \to 3$

$x(0) = 2\cos(0) = 2$

$x(1) = 2\cos\frac{\pi}{2} = 0$

$x(2) = 2\cos\pi = -2$

$x(3) = 2\cos\frac{3\pi}{2} = 0$



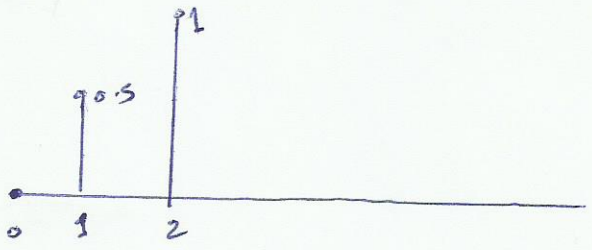
$h(n) = 0.5\,n\,[u(n) - u(n-3)]$

valid only at $n = 0 \to 2$

$h(0) = 0$

$h(1) = 0.5 \cdot 1 = 0.5$

$h(2) = 0.5 * 2 = 1$



$\therefore h(n) = 0.5\,\delta(n-1) + \delta(n-2)$

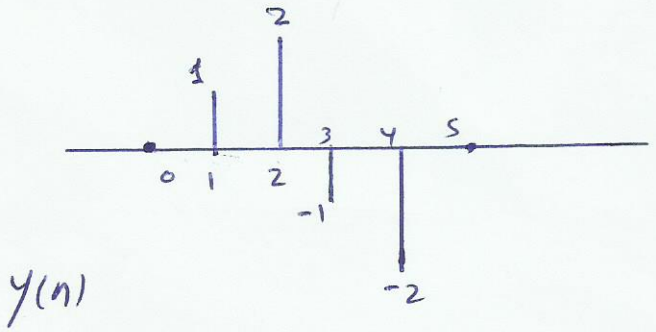
$\therefore y(n) = 0.5\,x(n-1) + x(n-2)$

Range

$$\begin{array}{r} 0 \quad 3 \\ 0 \quad 2 \\ \hline 0 \quad 5 \end{array}$$

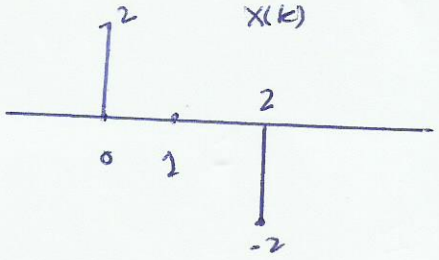
[5]

n	0	1	2	3	4	5
$0.5\,x(n-1)$	0	1	0	-1	0	0
$x(n-2)$	0	0	2	0	-2	0
$y(n)$	0	1	2	-1	-2	0

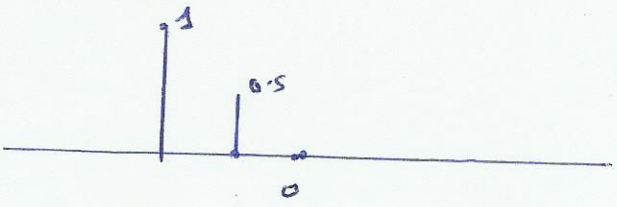


$y(n)$

by Drawing

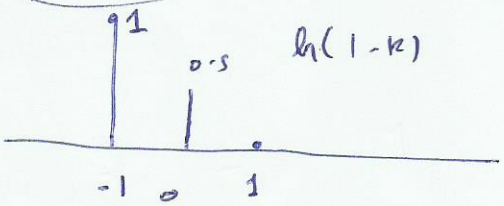


h(-k)



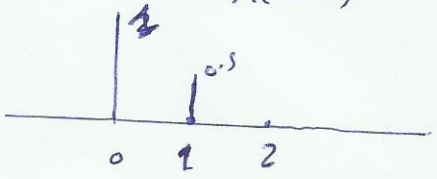
$y(0) = 0$

h(1-k)



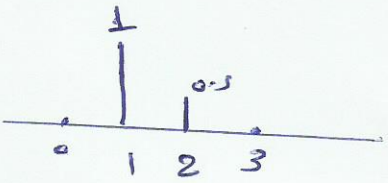
$y(1) = 1$

h(2-k)



$y(2) = 2 + 0 + 0 = 2$

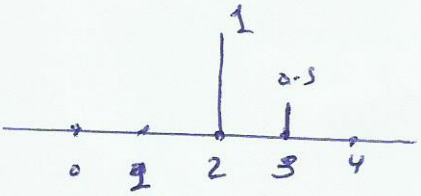
h(3-k)



$y(3) = 0 + 0 + (-1) + 0$

$y(3) = -1$

h(4-k)



$y(4) = 0 + 0 + (-2)$

$y(4) = -2$

$y(5) = 0$

3 (B)

7

$x(n) = (0.9)^n u(n)$

$h(n) = n\, u(n)$

4

Sol

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [(0.9)^k u(k)]\, [(n-k)\, u(n-k)]$$

$$= \sum_{0}^{n} (n-k)\, 0.9^k$$

$$= n \sum_{0}^{n} 0.9^k - \sum_{0}^{n} k\,(0.9)^k$$

$$= n\, \frac{1-(0.9)^{n+1}}{1-0.9} - \frac{n(0.9)^{n+2} - (n+1)\,0.9^{n+1} + 0.9}{(1-0.9)^2}$$